Minimum Of QCD Effective Action As Test Of QCD Confinement Parameter μ

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Abstract

Recently (hep-ph/0109278), a modification $A_\mu \rightarrow B_\mu = (1 + \mu\partial_m)A_\mu$ for the gluon field A_μ in the QCD Lagrangian (m = small gluon mass), yielded a k^{-4} behaviour for the gluon propagator, conventionally associated with linear confinement. This was prompted by a lack of consensus of results from other standard non-perturbative approaches on the T- dependence of gluon condensates in the cosmological context. Indeed, the gluon and quark condensates, and the pionic f_π, were all reproduced with a value of $\mu = 1GeV$.We now provide a basis to the confinement scale parameter μ by finding its formal relation to the standard QCD scale Λ_{qcd} via the minimality condition for the *integrated* effective action Γ, up to the critical 2-loop level, using the Cornwall-Jackiw formalism for composite operators. To that end we determine the mass function $m(p)$ via the Schwinger-Dyson equation (as a zero of the functional derivative of Γ w.r.t S'_F) as an input, and use the stationarity condition on Γ as function of μ and $\alpha_s(\mu)$ to obtain the ratio $\Lambda/\mu = 0.246$, in fair accord with hep-ph/0109278. Inclusion up to the two-loop level has proved crucial for this agreement. The prospects of a non-perturbative formulation of QCD in terms of the B-field for a more convergent strong interaction treatment (compared to A-field) are discussed.

Keywords: confinement scale, QCD effective action, minimality condition, B-field.

1 Introduction

Recently a modification of the QCD Lagrangian was proposed [1] so as to incorporate *linear* confinement in its operational mode, namely a k^{-4} behaviour for the gluon propagator. Such an approach was motivated from the observation that the results from the standard non-perturbative approaches on the T- dependence of gluon condensates in the cosmological context showed little consensus [1], leading us to look into the prospects of such modification in some detail. Now the feature of a *linear* confinement may be formally achieved by the defining a new (non-hermian) field B_μ, $B_\mu^\dagger$ related to the actual gluon field A_μ that appears in the QCD Lagrangian:

$$B_\mu = (1 + \mu\partial_m)A_\mu; \quad B_\mu^\dagger = A_\mu(1 - \mu\partial_m) \tag{1.1}$$

wherein the derivative is w.r.t. a small gluon mass m that goes to zero after differentiation. The parameter μ, by the very nature of its appearance, is suggestive of a (low

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energy) confinement scale which must be intimately related to the more fundamental QCD parameter Λ_{qcd} that is governed by RG theory, and is *not* to be regarded as any empirical quantity with an independent status per se. Namely, while μ is a dimensional parameter, it is not to be identified with the corresponding QCD parameter which obeys RG constraints. At this stage it is only an effective parameter whose identification with the universal Regge slope yields results consistent with QCD-SR and χPT for $T = 0$, by way of calibration.

However this theoretical issue was not pursued any further in [1] which was mainly concerned with the thermal behaviour of some key QCD parameters (gluon and quark condensates), as a possible means of accessing the quark-gluon plasma (QGP) phase in the cosmological context. From this (limited) perspective, μ was taken as a formally independent scale parameter to be determined from experiment. And it was subjected to the test of the same three QCD parameters (quark and gluon condensates, plus the pionic constant f_π) whose thermal behaviour was being investigated in [1], and it was found that these were all reproduced with $\mu = 1GeV$, while keeping Λ_{qcd} fixed at the more or less standard value of $200MeV$.

In this paper we seek to bridge a conceptual gap in the theoretical status of the μ parameter by finding its link with the more fundamental QCD scale parameter Λ_{qcd}. Now the new field B corresponds to a propagator which, suppressing the color and tensor indices for simplicity, has the form

$$< 0|A(x)(1-\mu\partial_m)(1+\mu\partial_m)A(y)|0 >= [1-\mu^2\partial_m^2]D_m(x-y) \tag{1.2}$$

where $D_m(x-y)$ is the lowest order gluon propagator with a small mass m that goes to zero after all operations have been performed. Eq.(1.2) corresponds in momentum space to the propagator

$$\Delta(k) = \frac{1}{k^2+m^2} + \frac{2\mu^2}{(k^2+m^2)^2} \tag{1.3}$$

which in the $m = 0$ limit gives rise to a sum of the usual one-gluon-exchange (o.g.e.) propagator k^{-2} and a new piece $2\mu^2 k^{-4}$ which corresponds to *linear* confinement. This last is checked simply by a Fourier transform of (1.3) to the r-representation in the *instantaneous* approximation. Indeed, in the instantaneous approximation ($t = 0$), the sum of the o.g.e. and confinement propagators transform in the r-representation, to a potential $V(r)$ which is obtained as a limit of $m = 0$ through the following steps:

$$\begin{aligned} V(r) &= \int d^3\mathbf{k}dk_0 \exp i\mathbf{k}.\mathbf{r} - k_0t[\frac{1}{\mathbf{k}^2+m^2} + \frac{2\mu^2}{(\mathbf{k}^2+m^2)^2}] \qquad (1.4)\\ &= \delta(t)\int d^3\mathbf{k}\exp i\mathbf{k}.\mathbf{r}[1-(\mu\partial_m)^2]\frac{1}{\mathbf{k}^2+m^2}\\ &= 4\pi^2\delta(t)[1-(\mu\partial_m)^2]\frac{e^{-mr}}{r}\\ &\rightarrow 4\pi^2\delta(t)[\frac{1}{r}-\mu^2 r] \end{aligned}$$

As a consistency check, the o.g.e. and confinement terms come with *opposite* signs in r-space, while coming with the *same* sign in k-space. (This derivation may be compared with that of Gromes [2]).

Now to relate the μ parameter to the QCD scale parameter Λ_{qcd} governed by RG theory, a most natural principle is that of minimality of the QCD effective action Γ. To that end, we shall make use of the Cornwall-Jackiw-Tomboulis (CJT) [3] formalism for composite operators, closely following the treatment of Miransky [4]. To recall the essential logic of the formalism, the minimality condition on $\Gamma(G, \phi)$ can be treated at two distinct levels. At the first (usual) level, setting its functional derivative $\partial_G \Gamma(G, \phi) = 0$, w.r.t. the quark's Green's function G gives rise to the Schwinger-Dyson equation (SDE), leading to the determination of the mass function $m(p)$. At a second (less conventional) level, the *integrated* effective action Γ w.r.t. the loop momenta may be regarded as an ordinary function of its input parameters, in particular μ, so that the minimality Γ w.r.t. μ should give its desired connection with the Λ_{qcd} parameter. In this (second level) determination, the (first level) SDE acts as the feeder in which the mass function $m(p)$ plays a central role, with $m(0)$ (related in some way to the *constituent* mass) expressed in terms of μ and $\alpha_s(\mu)$. We shall make this determination up to two-loop irreducible diagrams, and find that a rather big improvement over the one-loop value results from an inclusion of the 2-loop contribution.

In Sect.2, we sketch the CJT [3] formalism as given in Miransky [4], and set up the effective action up to the two-loop level in a notation closely following ref. [4], and obtain the mass function $m(p)$ as a solution of the SDE. In Sects.3 and 4, we calculate the integrated effective action for the one- and two-loop contributions respectively, using the results of Sect 2 for $m(p)$, as well as the dimensional regularization method of t'Hooft and Veltman [5]. Sect 5 gives our principal result, viz.,$\Lambda_{qcd} = 0.246\mu$, from the minimality of the integrated action. A short discussion follows on the prospects of an alternative formulation of QCD in terms of the B-field as a more efficient tool for capturing strong interaction effects than is possible with the A-field, vis-a-vis other approaches to non-perturbative QCD.

2 Effective Action For Composite Operators

We first summarize the results of the basic CJT [3] formalism for the effective action for composite operators, as enunciated by Miransky [4] (Chapter 8), with a view to adapting it to QCD, by incorporating the structure (1.3) of the full gluon propagator (including confinement). To fix the ideas, the effective action Γ is a functional of both the vacuum average $\phi_c(x) =< 0|\phi(x)|0 >$, and the propagator $G(x, y) = i < 0|T\phi(x)\phi(y)| >$ corresponding to ϕ (a generic name for the collection of fields in a given Lagrangian). The minimum for effective action is expressed by the zeros of the functional derivatives

$$\frac{\delta\Gamma}{\delta\phi_c(x)} = 0; \quad \frac{\delta\Gamma}{\delta G(x, y} = 0 \tag{2.1}$$

Now the functional Γ admits a loop expansion (up to 2-loops) of the form [3,4]

$$\Gamma(\phi_c, G) = S(\phi_c) + \frac{i}{2} Tr \ln G^{-1} + \frac{i}{2} Tr[D^{-1}G] + \Gamma_2(\phi_c, G) + C \tag{2.2}$$

$S(\phi_c)$ being the classical action, D the lowest order propagator, and Γ_2 the effective action in the 2-loop order. The Tr in each term stands for the summation over all the internal variables (spin, polarization), including integration over momenta. Since we shall be interested in translational invariant solutions, with ϕ_c being a constant, we may henceforth simplify the notation by droping this parameter. Further, we shall employ the notation of the 'effective potential' $V(G)$ which is merely the reduced effective action after taking out the 4D δ-function (i.e. the 4D volume element) from the latter, a la ref [4].

To adapt Γ to this simplified notation for QCD, which has two distinct fields, quarks (with full propagator S'_F), and gluons (with full propagator Δ), we may use the QCD version of the QED form given by eq.(8.57) of ref [4] in an obvious matrix notation, namely,

$$\Gamma(S'_F, \Delta) = iTr[\ln \frac{S_F}{S'_F} + \frac{S'_F}{S_F} - \frac{1}{2}\ln\frac{D}{\Delta} - \frac{1}{2}\frac{\Delta}{D}] + \Gamma_2(S'_F, \Delta) + C \tag{2.3}$$

where S_F and D are the unperturbed forms of the quark and gluon propagators respectively, and we have normalized the arguments in the respective logarithms w.r.t. their unperturbed values.

These functions, in momentum space, are defined as:

$$iS'_F(p) = \frac{m(p) - i\gamma.p}{m^2(p) + p^2}; \quad iS_F(p) = \frac{1}{i\gamma.p}; \tag{2.4}$$

$$\begin{aligned} i\Delta_{\mu\nu}(k) &= \frac{\delta_{\mu\nu} - k_\mu k_\nu/k^2}{(k^2 + m^2)}[1 + 2\mu^2/(k^2 + m^2)] \\ iD_{\mu\nu}(k) &= \frac{\delta_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 + m^2} \end{aligned} \tag{2.5}$$

Here we have taken the Landau gauge, for which the $A(p)$ function is unity [4, 6], so that the $B(p)$ function may be directly read as the mass function $m(p)$. The small quantity m in the gluon propagator tends to zero at the end, while the current mass of the (light) quark has been ignored. Then substituting (2.4-5) in (2.3), and

evaluating the traces a la ref [4], eq.(2.3) may be written as an 'effective potential' V as a sum of the one-loop (V_1) and two-loop (V_2) contributions as momentum integrals in Euclidean space. The one-loop contribution is

$$\begin{aligned} V_1(S'_F, \Delta) &= \int \frac{2d^4p}{(2\pi)^4}[-\ln\frac{1 + m^2(p)}{p^2} - 2\frac{p^2}{m^2(p) + p^2} \\ &\quad - \ln(1 + \frac{2\mu^2}{p^2 + m^2}) + (1 + \frac{2\mu^2}{p^2 + m^2})] \end{aligned} \tag{2.6}$$

The two-loop contribution, in which gluon line is inserted within a quark loop [c.f. fig 8.5 (a) of ref [4] in coordinate space, is (eq.(8.59) of ref.[4])

$$\Gamma_2 = \frac{g_s^2 F_1.F_2}{2}\int d^4x d^4y tr[S'_F(x, y)\gamma_\mu S'_F(y, x)\gamma_\nu \Delta_{\mu\nu}]$$

noting that a corresponding diagram with a gluon line joining two separate quark loops does not contribute [4]. The color factor $F_1.F_2$ has the value $(-4/3)$. Written out in momentum space in the same notation and normalization as above, the contribution to the two-loop effective potential becomes

$$V_2(S'_F, \Delta) = \frac{g_s^2 F_1.F_2.}{2}\int\int \frac{d^4p d^4k}{(2\pi)^8} Tr[S'_F(p)\gamma_\mu S'_F(p - k)\gamma_\nu \Delta_{\mu\nu}] \tag{2.7}$$

where the momentum space functions are given by (2.4-5). The evaluation of (2.7) for V_2 is described in Sect 3. In the remainder of this Section we show the evaluation of V_1 after summarizing and refining the results of [1] on $m(p)$ via the SDE.

2.1 SDE And The Structure of $m(p)$

From ref [1], the full SDE, including the confining interaction of Eq. (1.1), is:

$$m(p) = -3g_s^2 F_1.F_2[1-\mu^2\partial_m^2]\int \frac{-id^4k}{(2\pi)^4}\frac{m(p-k)}{(m^2+k^2)[m^2(p-k)+(p-k)^2]} \tag{2.8}$$

where $g_s^2 = 4\pi\alpha_s$, $F_1.F_2 = -4/3$ is the color Casimir; the Landau gauge has been employed [6], and $m = 0$ after differentiation. Our defense of the Landau gauge is essentially one of practical expediency, since this gauge usually offers the safest and quickest route to a gauge invariant result, even without a detailed gauge check, for there has been no conscious violation of this requirement at any stage in the input assumptions. For an approximate solution of this equation, we adopt the following strategy. As a first step, we replace the mass function inside the integral by $m(p)$. Then the method of Feynman for combining denominators with an auxiliary variable u and a subsequent translation $k \to k + pu$ yields an integral which can be treated [1] by the method of dimensional regularization (DR) [5] after the generalization $4 \to n$. The result before integration is

$$\begin{aligned} m(p) &= +4g_s^2\int\frac{d^nk}{(2\pi)^n}\int_0^1 du[1-\mu^2\partial_m^2] \\ &\qquad \frac{m(p)}{[m^2(p)u+m^2(1-u)+p^2u(1-u)]^2} \end{aligned} \tag{2.9}$$

the integral on the RHS of Eq. (2.9) may be carried out by using the following formulae:

$$\int\frac{d^np}{(2\pi)^n}\frac{1}{(ap^2+b)^\alpha} = \frac{(b\pi/a)^{n/2}}{(2\pi)^n b^\alpha}B(n/2,\alpha-n/2); \tag{2.10}$$

$$\int\frac{d^np}{(2\pi)^n}ln(ap^2+b) = -\frac{(b\pi/a)^{n/2}}{(2\pi)^n}\Gamma(-n/2) \tag{2.11}$$

The resulting function of n has a pole at $n = 4$ which should be subtracted a la ref [5] by putting $n - 4 = \epsilon$ and expanding in powers of ϵ. The final result after the operation of $\mu^2\partial_m^2$ and striking out $m(p)$ from both sides, is

$$\begin{aligned} \frac{\pi}{\alpha_s(\mu)} &= \int_0^1 du[\mu^2/\Omega(u) - \gamma - ln[\Omega(u)/\mu^2] \\ \Omega(u) &= m(p)^2u + m^2(1-u) + p^2u(1-u) \end{aligned} \tag{2.12}$$

An approximate solution of eq.(2.12) may be obtained with the replacement of $\Omega(u)$ by $<\Omega>= m(p)^2/2 + p^2/6$ in the $m = 0$ limit, when eq.(2.12) reduces to

$$z = x - \gamma + \ln x; \quad x \equiv \mu^2/<\Omega>; \quad z \equiv \pi/\alpha_s(\mu) \tag{2.13}$$

This is a transcendental equation in x, whose approximate solution is

$$x \approx z + \gamma - \ln(z+\gamma) + \frac{\ln(z+\gamma)}{z+\gamma} \equiv f(z) \tag{2.14}$$

Then an explicit solution for $m(p)$ is found from the last two equations as

$$m(p)^2 = 2\mu^2/f(z) - p^2/3 \tag{2.15}$$

A simpler solution which nevertheless incorporates the bulk (non-perturbative) effects, is obtained by neglecting the perturbative propagator, in which case eq.(2.14) reduces to $z = x$ only, so that the mass function acquires the simpler form

$$m(p)^2 = 2\mu^2/z - p^2/3 = m_q^2 - p^2/3; \quad m_q^2 \equiv 2\mu^2\alpha_s(\mu)/pi \tag{2.16}$$

which is a slight improvement over the corresponding result of [1].

3 Evaluation of 1-Loop Effective Potential V_1

We shall now use the result (2.16) for $m(p)$ to evaluate the effective potentials V_1 given by eq.(2.6), by the method of DR [5]. [The next section deals with the corresponding two-loop potential V_2, eq.(2.7)]. Denoting the integrals of Eq. (2.6) by V_{1i} $i = 1 - 4$, we first consider V_{11} to illustrate the steps of the DR method [5]. Thus we write

$$V_{11} = -\mu^{4-n} \int \frac{2d^n p}{(2\pi)^n} \ln(2/3 + m_q^2/p^2) \tag{3.1}$$

where we have as usual [1] supplied a compensating dimensional factor μ^{4-n} in front and have substituted from eq.(2.16). Using eq.(2.11), we now get

$$V_{11} = \frac{2\pi^2\mu^4}{(2\pi)^4}[\frac{3m_q^2}{2\mu^2}]^{n/2}\Gamma(-n/2) \tag{3.2}$$

where, in the π factors in front, we have set $n = 4$ [1], which may be regarded as coming under a 'modified' minimal subtraction scheme [4]. Setting $n = 4\epsilon$ and subtraction the $\epsilon = 0$ pole contribution [1], gives finally

$$V_{11} = +\frac{\pi^2\mu^4}{(2\pi)^4}y^2[3/2 - \gamma - \ln y]; \quad y \equiv 3m_q^2/2\mu^2 \tag{3.3}$$

Proceeding in an exactly similar way, the other terms in (2.6) may be evaluated.Thus

$$\begin{aligned} V_{12} &= -\mu^{4-n} \int \frac{4d^n p}{(2\pi)^n} \frac{p^2}{m^2(p) + p^2} \qquad (3.4) \\ &= -\frac{6\pi^2\mu^4}{(2\pi)^4}y^2[1 - \gamma - \ln y]; \quad y \equiv 3m_q^2/2\mu^2 \end{aligned}$$

$$\begin{aligned} V_{13} &= -\mu^{4-n} \int Tr \frac{2d^n p}{(2\pi)^4} \ln[1 + \frac{2\mu^2}{p^2 + m^2}] \qquad (3.5) \\ &= +\mu^4 \frac{\pi^2}{(2\pi)^4}[3/2 - \gamma - \ln 2] \end{aligned}$$

$$\begin{aligned} V_{14} &= \mu^{4-n} \int \frac{2\pi^n}{(2\pi)^n}(1 + \frac{2\mu^2}{p^2 + m^2}) \qquad (3.6) \\ &\Rightarrow ZERO \end{aligned}$$

the last integral vanishing on taking $m = 0$. This completes the evaluation of V_1.

4 Evaluation Of 2-Loop Effective Potential V_2

We now turn to the two-loop potential V_2 defined by (2.7) as a double 4D integral. A convenient strategy is first to integrate w.r.t. d^4p over the two fermionic propagators so as to give rise to a gluon self-energy operator. The second integral w.r.t. d^4k then gives a vacuum self-energy graph by joining up the 2 gluon lines. To organize the integral, we first define the gluon self-energy operator as

$$\Pi_{\mu\nu}(k) = \mu^{(4-n)} \int \frac{g_s^2 F_1.F_2 d^n p}{2(2\pi)^n} Tr[S'_F(p)\gamma_\mu S'_F(p-k)\gamma_\nu] \tag{4.1}$$

where S'_F is given by (2.4). Then

$$V_2 = \mu^{4-n} \int \frac{d^n k}{(2\pi)^n} \Pi_{\mu\nu}(k) \Delta_{\mu\nu}(k) \tag{4.2}$$

where $\Delta_{\mu\nu}$ is given by (2.5) in the Landau gauge. To evaluate (4.1), we substitute from (2.4) and (2.16), introduce the Feynman variable $0 \leq u \leq$ to combine the two denominators, take the traces, give a translation $p \to p + uk$ and drop the odd terms. Because of DR [5], one should expect gauge invariance to be satisfied automatically, were it not for the approximate solution (2.16) which militates against it. To meet this requirement we may still resort to the old-fashioned method [7,8] of 'gauge regularization' to extract the gauge invariant terms. The result of all these steps is the gauge invariant operator

$$\begin{aligned} \Pi_{\mu\nu}(k) &= (k^2\delta_{\mu\nu} - k_\mu k_\nu) \int_0^1 du u(1-u) \frac{9\pi^2}{(2\pi)^4} \\ &\times g_s^2 F_1.F_2 \frac{\Gamma(2-n/2)\mu^{4-n}}{[3m_q^2/2 + k^2 u(1-u)]^{(2-n/2)}} \end{aligned} \tag{4.3}$$

where we have also carried out the dimensional integral over $d^n p$ using the formula (2.10). Next we do DR [5] a la [1]. This gives

$$\frac{\Gamma(2-n/2)\mu^{4-n}}{[3m_q^2/2 + k^2 u(1-u)]^{(2-n/2)}} \Rightarrow -[\gamma + \ln \frac{k^2 u(1-u) + 3m_q^2/2}{\mu^2}] \tag{4.4}$$

Substitution of (4.3-4) and (2.5) in (4.2) gives on simplification the k- integral for V_2:

$$\begin{aligned} V_2 &= -3g_s^2 F_1.F_2 \frac{9\pi^2}{(2\pi)^4} \int_0^1 du u(1-u) \int \frac{d^n k}{(2\pi)^4} \\ &\times \mu^{(4-n)} [\gamma + \ln \frac{k^2 u(1-u) + 3m_q^2/2}{\mu^2}][1 + \frac{2\mu^2}{k^2 + m^2}] \end{aligned} \tag{4.5}$$

where the factor 3 in front comes from the simplification of the k_μ factors in the Landau gauge. To organize the integral, note first that the γ term does not contribute in the $m = 0$ limit. There are now two terms, V_{21} and V_{22}, associated with the non-perturbative (μ-term) and perturbative (1-term) contributions respectively. Both integrals may be carried out a la formula (2.11). The results are:

$$\begin{aligned} V_{21} &= -4g_s^2 \frac{9\pi^4}{(2\pi)^8} (3m_q^2 \mu^2) \int_0^1 du \\ &[\frac{2\mu^2 u(1-u)}{3m_q^2}]^{(2-n/2)} \frac{\Gamma(1-n/2)}{(n/2-1)}; \end{aligned} \tag{4.6}$$

$$V_{22} = -4g_s^2 \frac{9\mu^4\pi^4}{(2\pi)^8} \int_0^1 duu(1-u)\Gamma(-n/2)[\frac{3m_q^2}{2\mu^2 u(1-u)}]^{n/2} \tag{4.7}$$

We now need to do DR [5] on both these integrals just like in the pieces of V_1 above. The case of V_{21} which has a simple pole at $n = 4$, is straightforward and gives

$$V_{21} = -4g_s^2 \frac{9\pi^4}{(2\pi)^8}(3m_q^2\mu^2)[-\gamma + \ln\frac{2\mu^2}{3m_q^2}] \tag{4.8}$$

where we have carried out an elementary integration over u in the process. The other quantity V_{22} is somewhat different in structure from the others since the DR [5] can be effected only after the u-integration which leads to

$$V_{22} = -4g_s^2 \frac{9\pi^4}{(2\pi)^8}(3m_q^2/2)^2 \frac{\Gamma^2(2-n/2)\Gamma(-n/2)}{\Gamma(4-n)} \tag{4.9}$$

This is a new feature which shows up as a *doublepole* in $4 - n = \epsilon$, so that DR now involves subtraction of *both* the negative powers of ϵ before collecting the finite terms when $\epsilon \to 0$.The steps are facilitated by the following expansions (for small x) [4]:

$$\begin{aligned} \Gamma(x) &= x^{-1} - \gamma + \frac{x}{2}[\gamma^2 + \pi^2/6]; \\ \Gamma^3(1+x) &= 1 - 3\gamma x + 3x^2(\gamma^2 + \pi^2/6)/2 \end{aligned} \tag{4.10}$$

The result is

$$\begin{aligned} V_{22} &= -4g_s^2 \frac{9\pi^4}{(2\pi)^8}(3m_q^2/2)^2 \\ &\quad [-(3\gamma)\ln(y) + \ln^2(y)/2 - 3\gamma + 7/4 + \gamma^2/2 - \pi^2/12] \end{aligned} \tag{4.11}$$

where

$$y = \frac{3m_q^2}{2\mu^2}; \quad g_s^2 = 4\pi\alpha_s(\mu) = 4\pi^2 y/3 \tag{4.12}$$

the last one coming from the relation (2.16), viz., $m_q^2 = 2\mu^2\alpha_s(\mu)/\pi$. Substitution in (4.8) and (4.11) then gives

$$\begin{aligned} V_{21} &= 6Cy^2(\gamma + \ln y); \quad C = \frac{\pi^2\mu^4}{(2\pi)^4} \\ V_{22} &= 3Cy^3[(\gamma - 3)\ln y + \ln^2(y)/2 - 3\gamma + 7/4 + \gamma^2/2 - \pi^2/12] \end{aligned} \tag{4.13}$$

In the same notation we also record the expressions for V_{1i} from Section 3 as

$$\begin{aligned} V_{11} &= Cy^2(3/2\gamma - \ln y); \quad V_{12} = -6Cy^2(1 - \gamma - \ln y) \\ V_{13} &= C[3/2 - \ln 2 - \gamma]; V_{14} = ZERO \end{aligned} \tag{4.14}$$

5 Results and Discussion

We are now in a position to use the results of (4.13-14) to determine the relation of the confining parameter μ with the QCD scale parameter Λ_{qcd} by demanding the minimality of the total effective potential $F(y) = V_1 + V_2$ regarded as a function of the ratio y, while holding μ fixed. Namely, $F'(y) = 0$ which after factoring out the trivial solution $y = 0$, simplifies to

$$f(y) \equiv 2 + 22\gamma + 22\ln y + 3y[-3 + \gamma + (3\gamma - 8)\ln y + 1.5\ln^2 y - 1.911] = 0 \tag{5.1}$$

This yields the result

$$y \equiv \frac{3\alpha_s(\mu)}{\pi} = 0.475; \quad \alpha_s = \frac{2\pi}{9\ln(\mu/\Lambda_{qcd})} \tag{5.2}$$

which provides the desired connection

$$\Lambda_{qcd} = \mu(0.246) \tag{5.3}$$

This result may be compared with the input values used in [1], viz., $\mu = 1GeV$ and $\Lambda_{qcd} = 200MeV$, taken from the spectroscopic data [9]. Thus the theoretical value agrees with the empirical inputs to within about 20%. To see the effect of including the 2-loop effects, the value obtained from minimising the the one-loop potential only, viz.,

$$F_1(y) \equiv y^2(3/2 - \gamma - \ln y) - 6y^2(1 - \gamma - \ln y)$$

yields $\ln y = 0.4 - \gamma$, leading to the estimate

$$\Lambda_{qcd} = 0.4512\mu$$

which is more than double the input value [1,9]. Thus the inclusion of the two-loop contribution is crucial for self-consistency in the determination.

5.1 Significance of μ Parameter

One should now ask " what is the theoretical status of μ vis-a-vis Λ_{qcd}" ? For while the latter is well-rooted in RG theory which is structured on pQCD, the introduction of the former in a more or less ad hoc manner demands a formal placement within the QCD framework. A conservative view would be to regard μ as a sort of intermediate scale which controls the value of α_s in the strong-interaction regime of confinement. Indeed its modest value of ~ 0.5, eq.(5.2) corresponds precisely to such a regime, just as its (much smaller) values corresponding to the heavier masses of W, Z bosons are more appropriate to the electroweak regime. For a more formal basis to μ one needs to go back to a closer look at the connection (1.1) between the original gluon field A_μ and a new one B_μ, with an obvious convention that in the hermitian conjugated relation the derivative acts from right to left. While the total content of the QCD Lagrangian remains unaltered, the latter can in principle be reformulated in terms of the fields B_μ and $B_\mu^\dagger$ by writing A_μ as

$$2A_\mu = (1 + \mu\partial_m)^{-1}B_\mu + B_\mu^\dagger(1 - \mu\partial_m)^{-1} \tag{5.4}$$

And although the total QCD content remains the same, the emphasis on a B-field centred perturbative formalism clearly implies a more efficient incorporation of confinement effects than is usually possible in terms of the A-field, much like an improved convergence often achieved with a more efficient convergence parameter. A more concrete analogy is perhaps to the "dynamical perturbation theory" of Pagels and Stokar [10], which effectively incorporates a good deal of QCD information in its vertex structure, so that its "loop" diagrams can afford to be free from criss-cross gluon lines. So far in this paper we have considered only the lowest order approximation in the B-field which yields the propagator (1.3),expressed in the A-field basis, viz.,

$$\frac{1}{k^2}+\frac{2\mu^2}{k^4}$$

whose r-space version in the instantaneous approximation is represented by eq.(1.4). but it already accounts for a bulk of non-perturbative (confinement) effects, which reveals a clear advantage of this alternative description over that of the A-field. And while in this paper, the exercise has been confined merely to a consistency check on the value of the μ parameter via the minimality of the effective action up to 2-loop terms (to demonstrate that it is not a free parameter), the possibility of a more systematic approach to non-perturbative QCD in terms of the B_μ fields, with their associated Feynman diagrams etc, is clearly indicated.

We hasten to add that the present approach to confinement in QCD is only one of many such attempts since the inception of QCD with which this phenomenon has been intimately associated. Leaving out the most prominent candidate, viz., lattice QCD which is supposed to be almost the last word on the subject (but not easy to compare directly with analytical approaches), a few samples with obvious overlap with the physics of the present one are those dealing with the string structure of QCD, especially with a *vector* type confinement [11, 12], or the Seiberg-Witten theory of flux-tubes [13] which in turn has an obvious similarity with [11, 12]. A cross section of other interesting approaches which however have less similarity to the present one are domain-like structures with chiral-symmetry breaking [14], Kugo-Ojima confinement criterion in Landau gauge QCD [15], and Chiral Lagrangian with confinement from the QCD Lagrangian [16]. Unfortunately so far most of these approaches, including the present one [1], do not seem to have been developed to an extent big enough for a more detailed comparison of their respective effects at the observational level to be as yet possible. In the meantime the more time-honoured approaches like QCD-SR [17] and chiral perturbation theory [18] for the simulation of strong interaction effects which have already shown extensive evidence of flexibility in applications, are more amenable to comparison, as evidenced from the few results already found in [1]. On the basis of this limited comparison, we are optimistic that the present approach offers a viable alternative to the former, with the added advantage of an explicit incorporation of confinement in its basic formulation, but a more detailed formulation will be the subject of a subsequent communication.

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